3 – Analyzing Loss Exposures

**Objective**: Explain why data used in analyzing loss exposures should be relevant, complete, consistent, and organized.

**1 – Data Requirements for Exposure Analysis**

Loss exposure analysis is often based on probability and the statistical analysis of data. The statistical analysis of loss exposures starts with gathering sufficient data in a suitable form. Once the data has been collected, it can be subjected to a variety of probability and statistical techniques that are frequently used by risk professionals.

The most common basis of an analysis of current or future loss exposures is information about past losses arising from similar loss exposures. To accurately analyze loss exposures using data on past losses, the data should be relevant, complete, consistent, and organized.

**Relevant Data**

**To analyze current loss exposures based on historical data, the past loss data for the loss exposures in question must be relevant to the current or future loss exposures being analyzed (WC related, customer related).** Example; if you are trying to assess its auto physical damage loss exposures for the next 12 months, it may examine is auto physical damage losses for the last 4-5 years and then take into account any changes in the makeup of its fleet and the rate of increase for repair costs to determine potential losses for the next 12 months.

However, much of the data may no longer be relevant because of advances in auto engineering, different designs, materials, and systems. Therefore, data from past years may not be relevant to today’s loss exposures.

**Relevant data for property losses includes the property’s repair or replacement cost at the time it is to be restored**, not the property’s historical value (its original cost) or book value (its historical value minus accumulated depreciation). **For liability losses, the data should relate to past claims that are substantially the same as the potential for future claims being assessed**. Even relatively minor difference in the factual and legal bases of liability claims can produce substantially different outcomes and costs.

**Complete Data**

Obtaining complete data about past losses for a particular loss exposures often requires relying on others, both inside and outside the organization. What constitutes complete data depends largely on the nature of the loss exposure being considered.

**Having complete information helps to isolate the cause of loss of each loss.** Furthermore, having **complete data enables the risk professional to make reasonably reliable estimates of the dollar amounts of future losses**.

**Considering loss exposures related to employee injuries would require historical loss data to include information regarding Cause of loss, loss amounts, the employees’ experience and training, the time of day of the loss, the task being performed, and the supervisor on duty at that time.** *Similarly, complete data on a property loss to a piece of machinery would include the cost of repairing or replacing any damaged or inoperative machinery, the resulting net loss of revenue, any extra expenses, or any overtime wages paid to maintain production*.

**Consistent Data**

**To be valid for making risk management decisions about future losses, loss data must also be consistent in at least 2 respects.** If data is inconsistent in either respect, the future loss exposures could be significantly underestimated or overestimated.

**First the data must be collected on a consistent basis (same accounting methods) for all recorded losses.** *Loss data is often collected from a variety of sources, each of which may use different accounting methods (SAP vs GAAP, or estimates vs actual paid amounts, original cost vs replacement cost).* Consequently, data collected in this manner is likely to be inconsistent.

**Second, data must be expressed in constant dollars, to adjust for differences in price levels**. Differences in price levels will also lead to inconsistency. Two physically identical losses occurring in different years will probably have different values*. Inflation distorts the later loss, making it appear more severe because it is measured in less valuable dollars*.

**To prevent this distortion, historical losses should be adjusted (indexed – by year) so that loss data is expressed in constant dollars.** *To express data in constant dollars means that the amounts reported are comparable in terms of the value of goods and services that could be purchased in a particular benchmark year. Price indices are used to adjust data so that they are in constant dollars*.

Example, suppose losses were reported over a 4 year period. To convert the losses from 4 years ago to the current year (constant) values for comparison, multiply the losses from 4 years ago by 1.08 (assuming an 8% increase). The losses from 3 years ago would be multiplied by the % reflected in the price indices to bring them to current constant values for comparison (example 1.06, then the losses from 2 years ago by 1.03).

**Organized Data**

Even if relevant, complete, and consistent, data that is not appropriately organized will be difficult to use to identify patterns and trends that will help to reveal and quantify potential future loss exposures. Data can be organized in a variety of different ways, depending on which is most useful for the analysis being performed.

Example, listing losses for particular loss exposures by calendar dates may disclose patterns that detect seasonal losses. **An array of losses – amounts of losses listed in increasing or decreasing value** *– could reveal clusters of losses by severity and could also focus attention on large losses, which are often most important for insurance risk management decisions. Organizing losses by size is also the foundation for developing loss severity distributions or loss trends over time*.

**2 – Nature of Probability**

**Objective:** Describe the nature of probability with respect to theoretical and empirical probability and the law of large numbers.

The probability of an event is the relative frequency with which the event can be expected to occur in the long run in a stable environment. Determining the probability that a certain event will occur can be an important part of exposure analysis in the risk management process. **Probability distributions represents probability estimates for a particular set of circumstances and the probability of each possible outcome**.

**Theoretical Probability and Empirical Probability**

Any probability can be expressed as a fraction, a percentage, or a decimal. The probability of an event that is totally impossible is 0, and the probability of an absolutely certain event is 1.0. Therefore, the probabilities of all events that are neither totally impossible nor absolutely certain are greater than 0 but less than 1.0.

Probabilities can be developed either from theoretical considerations or from historical data. **Theoretical probability is probability that is based on theoretical principles rather than on actual experience**. Probabilities associated with events such as coin tosses or dice throws can be developed from theoretical considerations and are unchanging. Example, from a description of a fair coin toss or a dice throw, a person who has never seen either can calculate the probability of flipping a head or rolling a four.

For example, the probability that a 62 year old male will die in a particular year cannot be theoretically determined, but must be estimated by studying the loss experience of a sample of men aged 62. The empirical probabilities deduced solely from historical data may change as new data re discovered or as the environment that produces those events changes.

Empirical probabilities are only estimates whose accuracy depends on the size and representative nature of the samples being studied. In contrast, theoretical probabilities are constant as long as the physical conditions that generate them remain unchanged.

Although it may be preferable to use theoretical probabilities because of their unchanging nature, they are not applicable or available in most of the situations that insurance and risk management professionals are likely to analyze, such as *auto accidents or workers comp claims. As a result, empirical probabilities must be used.*

**Law of Large Numbers**

**Probability analysis is particularly effective for projecting losses in organizations that have (1) substantial volume of data on past losses and (2) fairly stable operations so that (except for price level changes) patterns of past losses presumably will continue in the future**. In organizations with this type of unchanging environment past losses can be viewed as a sample of all possible losses that the organization might suffer.

The larger number of past losses an organization has experienced, the larger the sample of losses that can be used in the analysis. Consequently, the forecasts of future losses are more reliable (consistent over time) because the forecast is based on a larger sample of the environment that produced the losses. This is the application of the law of large numbers.

The law of large numbers states that as the number of similar but independent exposures units increases, the relative accuracy of predictions about future outcomes also increases. In an example of a coin toss, if the theoretical probability of a head on a coin toss is 50%. Based on the law or large numbers, as the number of samples increase, the empirical frequency of a head on a coin toss will move closer and closer to 50%. The number of coin tosses most likely to result in an empirical frequency of 50% is 100.

*As an example of Theoretical v Empirical, suppose an urn holds 4 marbles. 1 of the marbles is red and 3 are black. Assume that the number of red or black marbles is not known. Estimate the theoretical probability of choosing a red marble on one draw (sample) from the urn by repeatedly sampling the marbles and replacing each in the urn after the sampling.*

*After 20 samples a red marble has been chosen 8 times, which yields an empirical frequency of 40% (8 divided by 20). However, this estimate is an accurate because the theoretical probability is 25%, given that only 1 of the 4 marbles is red.*

*According to the law of large numbers, the relative inaccuracy between the empirical frequency (40%) and the theoretical probability (25%) will decline, on average, as the sample size increase. That is as the number of samples increases from 20 to either 200 or 2,000, the empirical frequency of choosing a red marble gets closer and closer to 25%.*

**The law of large numbers has some limitations. It can be used to more accurately forecast future events only when the events being forecast meet all 3 of these criteria.**

* **The events have occurred in the past under substantially identical conditions and have resulted from unchanging, basic causal forces**
* **The events can be expected to occur in the future under the same, unchanging conditions**
* **The events have been, and will continue to be, both independent of one another and sufficiently numerous.**

**3 – Characteristics of Probability Distributions**

**Objective**: Describe the two requirements of any properly constructed probability distribution and the characteristics of discrete and continuous probability distributions.

After probabilities have been determined, probability distributions can be constructed. The information provided by probability distributions can be instrumental in analyzing loss exposures and making risk management decisions.

A properly constructed probability distribution always contains outcomes that are both mutually exclusive and collectively exhaustive, and it must also show the probabilities associated with each of the possible outcomes. *There are 2 forms of probability distributions: discrete and continuous.* A discrete probability distribution has a finite number of possible outcomes, and a continuous probability distribution has an infinite number of possible outcomes.

**Requirements of a Properly Constructed Probability Distribution**

**The first requirement of any properly constructed probability distribution is that it must contain a list of outcomes that are mutually exclusive and collectively exhaustive**. The flip of the coin, one flip will provide only one outcome. Therefore, those two outcomes are mutually exclusive. In addition, theses two outcomes are the only outcomes possible and therefore, are collectively exhaustive.

**The second requirement of any properly constructed probability distribution is that it must show the probabilities associated with each of the possible outcomes**. The “Number of Hurricanes Making Landfall in State X During One Hurricane Season” meets the requirement on its probability column, which shows the probabilities for each of the six possible outcomes. These probabilities total 1.0 (or 100%), which confirms that the outcomes are collectively exhaustive.

|  |  |
| --- | --- |
| Number of Hurricanes Making Landfall | Probability (%) |
| 0 | .300 |
| 1 | .350 |
| 2 | .200 |
| 3 | .147 |
| 4 | .002 |
| 5 + | .001 |
| Total Probability | 1.000 |

**Theoretical Probability Distribution Example**

Theoretical probability distributions (based on the theoretical principles rather than on actual experience) are seldom used in risk management, but they are helpful in understanding probability distributions. In a theoretical probability distribution, consider the probability distribution of the total number of points on one throw of 2 dice. One red and one green. There are 36 equally likely outcomes (green 1, red 1; green 1, red 2;… green 6, red 6). The dice rolling exhibit shows three alternate presentations of this probability distribution – a table, a chart, and a graph.

**The probability distribution has these characteristics**;

* **All possible outcomes are mutually exclusive and collectively exhaustive (the graph will indicate total probability of 1.0 meaning collectively exhaustive)**
* Eleven point values are possible (ranging from a total of two points to a total of 12 points)
* As the chart in the exhibit indicates, the probability of a total of two points is 1/36 because only one of the 36 possible outcomes (green 1, red 1) produces a total of two pints. Similarly, 1/36 is the probability o a total of twelve points
* *The most likely total point value, seven points*, has a probability of 6/36, represented in the table of outcomes by the diagonal southwest-northeast row of sevens*. When the outcomes are presented as a graph, the height of the vertical line above each outcome indicates the probability of that outcome*.

**Empirical Probability Distribution Example**

The auto-damage exhibit shows an empirical probability distribution estimated from historical data.

*This probability distribution has a vast number of possible outcomes*, consisting of dollar amounts of loss ranging from $0 to $25,000 or more**. Because it would be impossible to calculate a separate probability for each of the possible amounts of loss, the outcomes are divided into categories or bins** shown in column 1. To determine the empirical probabilities shown in column 3, the number of losses shown in column 2 for each size category is divided by the total number of losses. The probability distribution satisfies the requirements of a properly constructed probability distribution in these ways:

* The outcomes are mutually exclusive, as any given loss falls into one category, and the outcomes are collectively exhaustive, as the sum of the probabilities shown in column 3 is 100% or 1.
* Column 3 shows probabilities for all the outcomes as categorized in column 1 (bins)

**The empirical probability distribution for auto physical damage losses differs in two ways from the theoretical probability distribution of the dice rolls:**

* **The outcomes shown in column 1 of the auto physical damage exhibit (size categories of losses) are arbitrarily defined boundaries, whereas the outcomes of a roll of dice are specific and observable.**
* **While the maximum possible dice total is twelve, the largest size of auto physical damage losses ($25,000 +) has no evident upper limit**.

**Discrete and Continuous Probability Distributions**

Probability distributions come in two forms: discrete and continuous. Discrete probability distributions has a finite number of possible outcomes, while continuous probability distributions have an infinite number of possible outcomes.

**Discrete probability distributions are usually displayed in a table that lists all possible outcomes and the probability of each. These distributions are commonly used to analyze how often something will occur; that is, they are frequency distributions**.

Discrete probability distributions have a finite number of outcomes. In contrast, **continuous probability distributions have an infinite number of possible outcome values and are generally represented in one of two ways. Either as a graph or by dividing the distribution into a countable number of bins**.

The Continuous Probability Distributions exhibit shows two graphs of continuous probability distributions. In each graph, the possible outcomes are presented on the horizontal axis, and the probabilities of those outcome are shown on the vertical axis, labeled “Probability Density”. The height of the line or curse above an outcome indicates the probability of that outcome. In the exhibit a flat line above the interval 1 to 1,000, indicates that all the outcomes between 0 and 1,000 are equally likely. The number on the horizontal axis figure, could represent units of time, distance, weight, money, or other variables, The curve starts at a very low probability on the vertical axis an increases until it reaches a peak at 500 and then declines to a very los probability again at, 1,000, illustrates that the very low and very high outcomes are unlikely and that the outcomes around 500 are much more likely.

*The other way of presenting continuous probability distributions is to divide the distribution into a countable number of bins. Dividing the losses not bins makes the continuous probability distribution resemble a discrete probability distribution with several outcomes. This method enables the analyst to calculate the probability that an outcome will fall within a certain range of outcomes*. It will allow you to see 40% the losses that fall between $0-$5K. If the insured maintains a $5K deductible it would eliminate 40% of the losses.

**4 – Using Central Tendency**

**Objective**: Describe the measures of central tendency and how they can be used in analyzing loss exposures.

In analyzing a probability distribution, *the measures of central tendency represent the best guess as to what the outcome will be (like a weather forecast).* For example, if a manger asked an underwriter what the expected losses from fire would be for a store that the underwriter had insured, the underwriter’s best guess would be one of the measures of central tendency of the frequency distribution multiplied by one of the measures of tendency of the severity distribution. So, if the expected number of fires was 2, and each fire had an expected severity of $5K, the underwriter would expect $10K in losses.

Central Tendency – the single outcome that is the most representative of all possible outcome included within a probability distribution.

After determining empirical probabilities and constructing probability distributions, a risk professional can use central tendency to compare the probability distributions, analyze loss exposures, and make risk management decisions. Many probability distributions cluster around a particular value, which may or may not be in the exact center of the distribution’s range of values. The three most widely accepted measures of central tendency are the expected value or mean, the median and the mode.

**Expected Value**

**The expected value is the weighted average of all the possible outcomes of a theoretical probability distribution. The weights are the probabilities of the outcomes**. The outcomes of a probability distribution are symbolized as x1, x2, x3, … xn (xn represents the las outcome in the series), having respective probabilities of p1, p2,p3,… pn. The distribution’s expected value is the sum of (p1 x x1) + (p2 x x2) + (p3 x x3) +… (pn x xn). Calculating the Expected Value of a Probability Distribution –

The procedure for calculating the expected value applies to all theoretical discrete probability distributions. For continuous distributions, the expected value is also a weighted average of the possible outcomes. However, calculating the expected value for a continuous distribution is much more complex and not discussed here.

|  |  |  |  |
| --- | --- | --- | --- |
| Total Points –  Both Dice (x) | Probability (p) | P x X | Cumulative Probability  (sum of p’s) |
| 2 | 1/36 | 2/36 | 1/36 |
| 3 | 2/36 | 6/36 | 3/36 |
| **Total Expected Value** | **36/36 = 1** | **252/36 = 7.0** |  |

**Mean**

Probabilities are needed to calculate a theoretical distributions expected value. However*, when considering an empirical distribution constructed from historical data, the measure of central tendency is not called the expected value, it is called the mean*. **The mean is calculated by summing the value in ta data set and dividing by the number of values, often used by risk professionals as the single best guess for forecasting future events**. In other words, the mean is the numeric average. **Just as the expected value is calculated by weighting each possible outcome by its probability, the mean is calculated by weighted each observed outcome by the relative frequency with which it occurs.**

*For example,* if the observed outcomes values are 2,3,4,5,5,5,6,6, and 8, then the mean equals 4.8, which is the sum of the values, 48 divided by the number of values, 10. The mean is only a good estimate of the expected outcome if the underlying conditions determining those outcomes remains constant over time.

*Unlike the expected value, which is derived from theory, the mean is derived from experience. If the conditions that generated that experience have changed, the mean that was calculated may no longer be an accurate estimate of central tendency.* Nonetheless, risk professionals often use the mean as the single best guess for forecasting future events.

The best guess as to the number of workers compensation claims that an organization will suffer in the next year is often the mean of the frequency distribution of workers compensation claims from previous years.

**Median and Cumulative Probabilities**

Another measure of central tendency is the median**. Median is the value at the midpoint of a sequential data set with an odd number of values or the mean of the two middle values of a sequential data set with an even number of values.**

To determine a data set’s median, its values must be arranged by size, from highest to lowest or lowest to highest. In the array of nineteen auto physical damages losses in the exhibit, the median loss has an adjusted value of $6,782. This tenth loss is the median because nine losses are greater, and nine losses are lower.

A probability distribution’s median has a cumulative probability of 50%. In the rolling of the dice example the probability distribution, 7 is the only number of points for which the probability of outcomes (15/36) is equal to the probability of lower outcomes (15/36). That is, there are 15 equally probable ways of obtaining an outcome higher than 7 and 15 equally probable ways of obtaining an outcome lower than 7.

The median can also be determined by summing the probabilities of outcomes equal to or less than a given number of points. The cumulative 50% probability (18/6) is reached in the 7 point category. Therefore, 7 is the median of this distribution.

The cumulative probabilities in column 4 of the exhibit indicate the probability of a die roll yielding a certain number of points or less. For example, the cumulative probability of roll a three or less is 3/36 (or the sum of 1/36 for rolling a two and 2/36 for rolling a three) Similarly the cumulative probability of rolling a ten or less is 33/36, calculated by summing the individual column 2 probabilities of outcomes of ten points or less.

**When probability distributions of losses, calculating probabilities of losses equal to or less than a given number of losses or dollar amounts of losses, individually and cumulatively *(median), can be helpful in selecting retention levels.* Similarly, calculating individual and cumulative probabilities of losses equal to or greater than a given number of loses or dollar amounts *can help in selecting upper limits of insurance coverage****.*

The “Cumulative Probabilities” exhibit shows how to derive a cumulative probability distribution of loss sizes from the individual probabilities of loss size in the exhibit. Column 3 indicates that, on the basis of the available data, 36.84% of all losses are less than or equal to $5K and that another 36.84 are greater than $5K but less than or equal to $10K. Therefore, the probability of a loss being $10K or less is calculates as the sum of these two probabilities, or 73.68% (column 4). Similarly, as shown in column 7, individual losses of $10K or less can be expected to account for 41.03 % of the total dollar amount of all losses.

Understanding the cumulative probability distribution will enable and analyst to evaluate the effect of various deductibles and policy limits on insured loss exposures. If an insurance policy has a $5K deductible, the analyst would know that 36.84% of losses covered by that policy would be below the deductible level and therefore would not be paid by the insurer.

The summed probabilities in column 4 exhibit indicate that the median individual loss is between $5,001 and $10,000 the category in which the 50% cumulative probability is reached

**Mode**

**Mode is the most frequently occurring value in a distribution. It enables risk professionals to focus on the outcomes that are the most common**.

A further measure of central tendency is the mode. In a continuous probability distribution graph, the mode is the value of the outcome directly beneath the peak of the curve. In the distribution of total points of two dice throws the mode is seven points. In the empirical distribution of auto physical damage losses, the mode is the $0-$5K range or the $5,0001 - $10K range, because those ranges each have the highest frequency of losses (seven).

**Knowing the mode of a distribution allows the analyst to focus on the outcomes that are most common. Knowing that the most common auto physical damage losses are in the $1-$10K range *may influence decision regarding deductible levels for insurance coverage****.*

The relationships among the mean (average), median, and mode for any data set are illustrated by the distribution’s shape. The shape of a particular frequency or severity probability distribution can be seen by graphing the data as shown “typical shapes” and can be either symmetrical or asymmetrical and skewed distributions.

In a symmetrical distribution, one side of the curve is a mirror image of the other. The distribution reflected in a bell-shaped figure, or a full arch in another figure. The mode has the same value s that of the mean and median.

*If the distribution is asymmetrical, it is skewed. Skewed distributions are shown almost like a wave rising or ebbing. Many loss distributions are skewed because the probability of small losses is large whereas the probability of large losses is small. Asymmetrical distributions are common for severity distributions where most losses are small but there is a small probability of a large loss occurring.* If the distribution is skewed, the mean and the median values will differ, and the median value of the distribution is often a better guess than the mean as to what will most likely occur.

**5 – Using Dispersion**

**Objective**: Explain how the standard deviation and the coefficient variation measure dispersion in a probability distribution

When analyzing probability distributions, risk professionals can use measures of dispersion to assess the credibility of the measures of central tendency used in analyzing loss exposures.

Measures of central tendency for a distribution of outcomes include the expected value or the mean, which can help risk professionals make best guesses as to which outcome in a probability distribution will occur. Another important characteristic of distribution is its dispersion**. Dispersion describes the extent to which the distribution is spread out rather than concentrated around the expected value or the mean (the variation among values in a distribution)**. The less dispersion around the distribution’s expected value or mean, the greater likelihood that actual results will fall within a given range of expected value or mean.

Therefore, less dispersion means less uncertainty about the expected outcomes. An underwriter may be ale to use measures of dispersion around estimated losses to determine whether to offer insurance coverage to an applicant for insurance. Dispersion also affects the shape of a distribution when it is graphed. A more dispersed distribution forms a flatter curve, whereas a less dispersed distribution forms a more peaked curve. There are two widely used statistical measures of dispersion: the standard deviation and the coefficient of variation.

**Standard Deviation (will not be asked to calculate this)**

The standard deviation is the average of the differences (deviations) between the values in a distribution and the expected value (or mean) of that distribution. The standard deviation therefore indicates how widely dispersed the value in the distribution are. Used to assess the credibility of Loss Runs.

To calculate the standard deviation of a probability distribution, one must perform these steps:

1. Calculate the distributions expected value or mean
2. Subtract this expected value from each distribution value to find the differences
3. Square each of the resulting differences
4. Multiply each square by the probability associate with the value
5. Sum the resulting products
6. Find the square root of the sum

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Calculation of | Standard | Deviation of | The Probability | Distribution | Two dice |
| (1) points (x) | (2) Probability (p) | (3) Step 1  EV | (4) Step 2  X1 – EV | (5) Step 3  (X1 – EV)2 | (6) Step 4  (X1 – EV)2 x P |
| 2 | 1/36 | 7 | -5 | 25 | 25/36 |
| 3 | 2/36 | 7 | -4 | 16 | 32/36 |
| 4 | 3/36 | 7 | -3 | 9 | 27/36 |
| 5 | 4/36 | 7 | -2 | 4 | 16/36 |
| 6 | 5/36 | 7 | -1 | 1 | 5/36 |
| 7 | 6/36 | 7 | 0 | 0 | 0 |
| 8 | 5/36 | 7 | 1 | 1 | 5/36 |
| 9 | 4/36 | 7 | 2 | 4 | 16/36 |
| 10 | 3/36 | 7 | 3 | 9 | 27/36 |
| 11 | 2/36 | 7 | 4 | 16 | 32/36 |
| 12 | 1/36 | 7 | 5 | 25 | 25/36 |
|  |  |  | Step 5 | Total | 210/36 |
|  |  |  | Step 6 | Square(210/36) | 2.42\* |
| Rounded \* |  |  |  |  |  |

To calculate the standard deviation using the actual sample of outcomes, it is not necessary to know the probability of each outcome, just how often each outcome occurred.

**The steps for calculating the standard deviation of a set of individual outcomes not involving probabilities are these steps:**

1. **Calculate the mean of the outcomes (the sum of the outcomes divided by the number of outcomes)**
2. **Subtract the mean from each of the outcomes**
3. **Square each of the resulting differences**
4. **Sum these squares**
5. **Divide this sum by the number of outcomes minus one (this value is called the variance)**
6. **Calculate the square roof of the variance**

The calculation illustrates how to calculate a standard deviation using actual loss data rather than theoretical probability distribution. Risk professionals use measurements of dispersion of the distributions of potential outcomes to gain a better understanding of the loss exposures being analyzed.

For example, knowing the expected number of workers compensation claims in a given year is important, but it is only one element of the information that can be gleaned from a distribution. The standard deviation can be calculated to provide a measure of how confident a risk professional can be in his or her estimate of the expected number of workers compensation claims.

Calculation of Standard Deviation of Individual Outcomes

|  |  |  |  |
| --- | --- | --- | --- |
| (1)  Adjusted Loss Amount (ALA) | (2)  Step 1  Mean Loss (ML) | (3)  Step 2  ALA – ML | (4)  Step 3  (ALA – ML) Squared |
| $ 200 | $8,909 | $ -8,709 | $ 75,846,681 |
| 1,300 | 8,909 | * 7,609 | 57,896,881 |
| 1,500 | 8,909 | -7,409 | 54,893,281 |
| 2,300 | 8,909 | -6,609 | 43,678,881 |
| 4,000 | 8,909 | -4,909 | 24,098,281 |
| 4,224 | 8,909 | -4,685 | 21,949,225 |
| 4,483 | 8,909 | -4,426 | 19,589,476 |
| 5,500 | 8,909 | -3,409 | 11,621,281 |
| 5,999 | 8,909 | -2,910 | 8,468,100 |
| 6,782 | 8,909 | -2,127 | 4,524,129 |
| 7,402 | 8,909 | -1,507 | 2,271,049 |
| 8,303 | 8,909 | -606 | 367,236 |
| 8,403 | 8,909 | -506 | 256,036 |
| 9,059 | 8,909 | 150 | 22,500 |
| 13,599 | 8,909 | 4,690 | 21,996,100 |
| 13,699 | 8,909 | 4,790 | 22,944,100 |
| 15,589 | 8,909 | 6,680 | 44,622,400 |
| 21,425 | 8,909 | 12,516 | 156,65,256 |
| 35,508 | 8,909 | 26,599 | 707,506,801 |
| **Step 4** | **Sum** |  | **$1,279,202,694** |
| **Step 5** | **Variance** | **(sum divided by (n – 1)**  **(# of outcomes -1 = 18)** | **$71,066,816** |
| **Step 6** | **Standard Deviation** | **(square root variance)** | **$8,430** |

**Coefficient of Variation**

The coefficient of variation is a further measure of dispersion of a distribution. Coefficient of variation is a measure of dispersion calculated by dividing a distribution’s standard deviation by its mean**. Coefficient of variation is used to compare two distributions with different means**. For example, the coefficient variation of the distribution of total points in rolling two dice equals 2.4 points (the standard deviation of the distribution) divided by 7.0 points (the mean or expected value), which is 0.34. Similarly, the coefficient of variation of the sample of outcomes in the (above table) is $8,430 divided by $8,909 (mean), or approximately 0.95.

**In comparing the two distributions if both distributions have the same mean (or expected value), then *the distribution with the larger standard deviation has the greater variability*. If the two distributions have different means (or expected values), the coefficient of variation is often used to compare the two distributions to determine which has the greater variability relative to its mean (or expected value).**

For a simplified example, consider an underwriter who can accept only one of two submissions for insurance. *Each submission includes a severity distribution of the applicant’s losses incurred over the past five years. In the unlikely event that both distributions have the same mean, the underwriter should, all other things being equal,* ***accept the submission with the lower standard deviation****. If, as is more likely, the two distributions have different means and standard deviations, the underwriter should, all things being equal,* ***accept the submission with the lower coefficient of variation***.

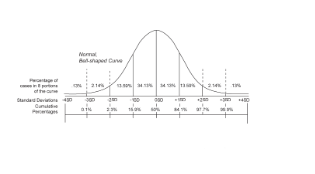
**6 – Using Normal Distributions**

**Objective**: Describe the characteristics of normal distributions and how they can be used in risk management.

**A normal distribution is a probability distribution that can help risk professionals evaluate the variability around the mean and has therefore proven useful in risk management and insurance. A normal distribution is a probability distribution that, when graphed, generates a bell shaped curve.**

**Characteristics of Normal Distributions**

In a bell-shaped cure of a normal distribution. Note that the normal curve never touches the horizontal line at the base of the diagram. *In theory, the normal distribution assigns some probability greater than zero for every outcome, regardless of its distance from the mean*.



**34.13% + 34.13%** = 68.26%

13.59% + (68.26%) **+ 13.59%** = 95.44%

2.15% + (95.44%) **+ 2.15%** = 99.74%

For example, **for all normal distributions, a 34.13% of all outcomes are within one standard deviation above the mean and, because every normal distribution is symmetrical, another equal 34.13% of all outcomes fall within one standard deviation below the mean**. By addition, *68.26% of all outcomes are within one standard deviation above or below the mean*. The portion that is between one and two standard deviations above the mean contains 13.59% of all outcomes, as does the portion between one and two standard deviations below the mean. Hence the area between the mean and the two standard deviations above the means 47.72% (34.13% + 13.59%) of the outcomes and another 47.72% are two standard deviations or less below the mean.

**95.44% of all outcomes are within two standard deviations above or below the mean**, and fewer than 5% of outcomes are outside the two standard deviations above or below the mean.

Consequently, the portion of the distribution between three standard deviations above the mean and three standard deviations below it contains 99.74% of all outcomes. Therefore, *only 0.26% (100 – 99.74) of all outcomes lie beyond three standard deviations from the mean. Half of these outcomes (0.13) are more than three standard deviations below the mean, and the other half (0.13) are more than three standard deviation above the mean.*

In a Normal Distribution all of the following are TRUE:

1. 50% probability that a data point will be above or below the mean;
2. 68% probability that a data point will be plus or minus one standard deviation of the mean; 34% (1/2) that the data point will be one below the mean, and another 34% above the mean;
3. 95% chance +/- the data point will be 2 standard deviations from the mean
4. 99.7% chance that a data point will be within 3 standard deviations.

**Practical Application**

**The relationship between the expected value and the standard deviation of a normal distribution can have useful practical application. The characteristics of the expected value and standard deviation of a normal distribution can help management select an acceptable probability for loss and aid in scheduling maintenance or selecting retention levels on various loss exposures**.

For example, suppose that a manufacturer uses 600 electrical elements to heat rubber. The useful life of each element is limited, and an element that is used for too long poses a substantial danger of exploding and starting an electrical fire. An insurance professional underwriting the manufacturer’s commercial property insurance would look for evidence that proper maintenance is performed, and the elements are replaced periodically to ensure proper fire safety.

The issue is determining when to replace the elements. Replacing them too soon can be costly, whereas replacing them too late increases the chance of fire. The characteristics of the normal probability distribution provide a way of scheduling maintenance so that the likelihood of an element becoming dangerous before it is replaced can be kept below a particular margin of safety that is specified by the organization based on its willingness to assume risk.

Assume the expected safe life of each element conforms to a normal distribution having a mean of 5,000 house and a standard deviation of 300 hours. Even if the maintenance schedule requires replacing each element after it has been in service only 5,000 hours (the mean, or expected safe life**), a 50% chance exists that it will become unsafe before being changed, because 50% of the normal distribution is below this 5,000-hour mean**.

If each element is changed after having been used only 4,700 **hours (one standard deviation below the mean** (5,000 – 300), a 15.87% **(50% - 34.13**) chance still exists that an element will become unsafe before changing it. If this probability of high hazard is still too high, changing the element after 4,400 hours **(two standard deviations below the mean)** reduces the probability of high to only **2.28%,** the portion of a normal distribution that is more than two standard deviations below the mean.

A still more cautious practice would be to change elements routinely after only 4,100 hours (**three standard deviations below the mean** (5,000 – (3 x 300), so that the probability of an element becoming highly hazardous before replacement would be only **0.13%,** slightly more than one in 1,000 chance.

Using this analysis, management can select an acceptable probability that an element will become unsafe before being replaced and can schedule maintenance accordingly.

Practice Exercise: An insurer is beginning to write policies in a new state. The insurer’s claim manger wants to know how many new claim representatives to hire. The insurer is marketing department has provided an estimate of additional premium volume from the new state. Based on that estimate and industry data, the manager has determined the mean *number of new claims to be 8,000* *with a standard deviation of 2,000 in a normal distribution*. If a claim representative *can adjust 600 claims* per year and the manager wants to be approximately 98% certain that she has enough representatives, how many will she need to hire?

The Normal distribution – Percentages of Outcome Within Specified Standard Deviations of the Mean, 2.28% of all outcomes (2.15% + 0.13%) are more than two standard deviations above the mean, and 97.72 (100% - 2.28%) of all outcomes fall under the normal distribution below two standard deviations above the mean, and 97.72 above the mean. Therefore, by rounding up the 97.72%, the claim manager can be approximately certain that the actual number of claims will fall at or below two standard deviations above the mean is 12,000 claims (calculated as 8,000 + 2,000 + 2,000). Because each claim representative can adjust 600 claims per year, the manager will need to hire 12,000/600 or 20 new representatives.

Mean = 8,000 Deviation = 2,000

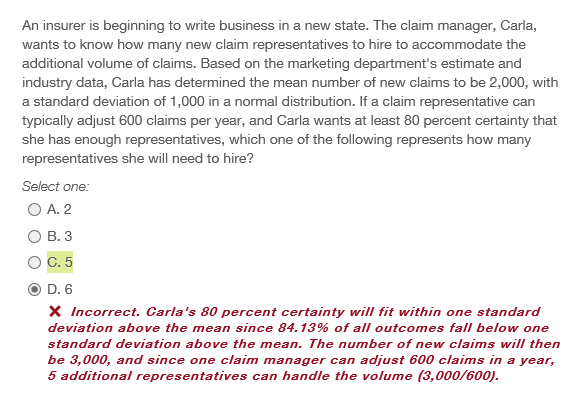
in this case we use the Mean $8,000 add the losses probabilities one standard below the median 2,000 and one standard above the median 2,000 totaling 12,000 divided by the 600 claims per year and you will need to hire 20 new claim representatives.

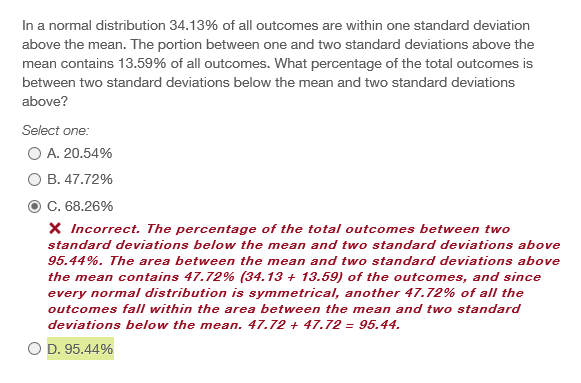
Assume that ABC Manufacturing’s total losses per year are normally distributed. The average mean of the firm’s losses is $500,000 and the standard deviation is $40,000. Assuming underlying conditions do not change, what is the probability that its losses next year will be between $460,000 and $540,000? What is the probability that its losses next year will be between $500,000 and $540,000?

The probability that ABC manufacturing’s loss probability would be between 460,000 and 540,000 is 68.26%. The probability that ABC Manufacturing’s loss probability would be between $500,000 and $540,000 is 34.13%

Mean= $500,000 Deviation = $40,000

34.13 deviation below $500,000 = 460,000 add the 34.13 to the loss above the deviation gives you the 68.26%





**7 – Analyzing Loss Exposures**

**Objective:** Explain how to analyze loss exposure considering the four dimensions of loss and data credibility.

Analyzing loss exposures enables the risk analyst to develop loss projections that will guide the analyst in prioritizing loss exposures and selecting appropriate risk management techniques to mange the loss most effectively.

The analysis step of the risk management process, also known as risk assessment, involves considering the **four dimensions of loss exposure: loss frequency, loss severity total dollar losses, and timing**. In addition, the credibility of the data used to project these dimensions must be determined**. Data credibility is the level of confidence that available data is an accurate indication of future losses.**

**Loss Frequency**

**Loss frequency is the number of losses that occur during a specific period. Relative loss frequency is the number of losses that occur within a given period relative to the number of exposure units (such as number of buildings or cars exposed to loss).**

If an organization experiences, on average, five theft losses per year, five is the mean of an empirical frequency distribution (empirical is based on actual experience). If the organization has only one building, then both the loss frequency and the relative frequency of losses from theft is 5 per year. However, if the organization has 5 buildings, then the organization still has a loss frequency of 5 theft losses per year, but the relative frequency is one loss per year per building. Two of the most common applications of relative frequency measures in risk management are injuries per employee per hour worked and auto accidents per mile driven.

Frequency distributions are usually discrete probability distributions based on past data regarding how often similar events have happened. Example, the exhibit contains a frequency distribution of the number of hurricanes that make landfall in a fictious state during a single hurricane season. One way of describing the frequency of hurricanes is to report a mean frequency of occurrence, such as approximately 1.2 hurricanes making landfall per year.

However, the exhibit does not incorporate some of the other information available from the entire frequency distribution. For example, the most likely outcome may be one hurricane per year (35.0 % of the time). However, having 0 hurricanes per year is also reasonably likely (30% of the time), but having 5 or more hurricanes make landfall is reasonably unlikely (0.1% of the time). Therefore, an insurance risk management professional should supplement the mean of 1.2 with other information from the frequency distribution, such as the standard deviation (which is approximately 1.04) and skewness measures.

*Loss frequency can be projected with a fairly high degree of confidence for some loss exposures in large organizations*. A company that ships thousands of parcels each day probably can more accurately project the number of transit losses it will sustain in a year, based on past experience and adjuster for any expected changes in future conditions, than can a company that ships only hundreds of parcels each month.

*Most organization do not have enough exposures units to accurately project low frequency, high severity events (such as employee deaths). However, an estimate with a margin for error is better than no estimate as all, as long as its limitations are recognized*.

**Loss Severity**

The purpose of analyzing loss severity is to determine how serious a loss might be. How much of a building could be damaged in a single fire, how long might it take the business to resume operations after a fire loss?

**Maximum Possible Loss**

Effectively managing risk requires identifying the worst possible outcome of a loss. **The maximum possible loss (MPL) is the total value exposed to loss at any one location or from any one event**. In the case of a fire, the maximum possible loss is typically the value of the building plus the total value of the building’s contents.

To determine MPL for multiple exposures units, such as a fleet of cars, one should consider factors such as whether multiple vehicles travel together (a circumstance that could cause one vent, such as collision, to damage several vehicles) or whether multiple vehicles are stored in the same location (a circumstance that could cause one event, such as flood, hail, tornado, to affect several vehicles). This helps determine the maximum number of vehicles that could be involved in any one loss, the event’s MPL.

*Maximum possible loss can be estimated based on values exposed to loss, this estimation is not necessarily appropriate or possible for assessing maximum possible liability loss. In theory, liability losses are limited only by the defendant’s total wealth. Therefore, some practical assumptions must be made about the MPL in liability cases to properly assess that loss exposure*. Instead of focusing on the defendant’s total wealth, to liability loss 95% (or 98%) of the time in similar cases is the MPL.

**Frequency and Severity Considered Jointly**

To fully analyze the significance of a particular loss exposure, the analyst must consider both severity and frequency and how the interact**. One method of jointly considering frequency and severity is the Prouty Approach. Another method involves combining frequency and severity distributions to create a single total claims distribution.**

**The Prouty Approach**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Loss** | **Frequency** |  |  |
|  | **Almost Nil** | **Slight** | **Moderate** | **Definite** |
| **Severe** | Reduce or Prevent /Transfer | Reduce or Prevent  /Transfer | Reduce or Prevent  /Retain | Avoid |
| **Significant** | Reduce or Prevent /Transfer | Reduce  /transfer | Reduce or Prevent  /Retain | Avoid |
| **Slight** | Reduce or Prevent  /Retain | Reduce  /Retain | Reduce or Prevent  /Retain | Prevent  /Retain |

**As shown in the exhibit, the Prouty Approach entails 4 categories of loss frequency;**

* **Almost Nil – extremely unlikely to happen; virtually no possibility**
* **Slight – could happen but has not happened**
* **Moderate – happens occasionally**
* **Definite – happens regularly**

**There are three categories of loss severity**

* **Slight – organization can readily retain each loss exposure**
* **Significant – organization cannot retain the loss exposure, some part of which must be financed**
* **Severe – organization must finance virtually all of the loss exposure or endanger its survival**

These broad categories of loss frequency and severity are **subjective**. One organization may view losses that occur once a month as moderate, while another would consider such frequency as definite. Similarly, one organization may view $1M loss as slight, while another may view it as severe. *Theses categories can help insurance and risk management professionals prioritize loss exposures*.

A loss exposure’s frequency and severity tend to be inversely related. That is, the more severe a loss tends to be, the less frequently it tends to occur. Conversely, the more frequently a loss occurs because of a particular exposure, the less sever the loss tends to be.

Losses that generate minor but definite losses are typically retained and incorporated into the organization’s budget. At the other extreme, loss exposures that generate intolerably large losses are typically avoided. Most risk management decision about adopting risk control and risk financing techniques concern loss exposures for which individual losses although tolerable, tend to be either significant or severe and have a moderate, slight, or almost nil chance of occurring.

A given loss exposure might generate financially significant losses because of either high individual loss severity or high frequency, low severity losses that aggregate to a substantial total. Organization may be tempted to focus on high profile “shock events” such as a major fire, a hurricane, or a huge liability claim. However, smaller losses which happen so frequently that they become routine, can eventually produce much larger total losses than a single dramatic event. Minor cumulative significant losses usually deserve as much risk management attention as large individual losses.

**Another way of jointly considering frequency and severity is to combine both frequency and severity distributions into a total claim distribution, which can provide additional information about potential losses that may occur in a given period**. Combining distributions can be difficult because as the number of possible outcomes increases, the possible combinations of frequency and severity grow exponentially.

The exhibit presents a simple example of three possible frequencies (0, 1, and 2) and three possible severities ($100, $250, and $500) that represent shop lifting losses from a hardware store. The frequency and severity distributions for a given year are shown in the exhibit, along with the total claims distribution created by considering all the possible combinations of the frequency and severity distributions.

**Total Claims Distribution**

|  |  |  |  |
| --- | --- | --- | --- |
| Dollar Losses | Probability | Probability Calculation |  |
| $0 | .33 | P(f0) | There is only one possible way to have $0 losses: the frequency is 0 |
| 100 | .11 | P(f1) x P(s1) | There is only one possible way to have $100 in losses; one $100 loss |
| 200 | .04 | P(f2) x P(s1) x P(s1) | There is only one possible way to have $200 losses; two $100 losses |
| 250 | .11 | P(f1) x P(s2) | There I only one possible way to have $250 in losses; one $250 loss |
| 350 | .07 | P(f2) x P(s1) x P(s2)  P (f2) x PS2) x P(s1) | There are two possible ways to have $350 in losses; one $100 loss and one $250 loss, or one $250 loss and one $100 loss |

For example, a 33% chance exists of a loss not occurring during the year (frequency = 0). Therefore, in the total claims distribution, a 3% chance exists of the total losses being $0. There is only one possible way for a $100 dollar loss to occur; a frequency of 1 and a severity of $100. Therefore, that probability is .11 [.33 frequency 1) x .33 (severity of $100) = .11]

A total claim distribution can be used to calculate the measures of central tendency and dispersion and evaluate the effect that various risk control and risk financing techniques would have on this loss exposure.

**Total Dollar Losses**

The third dimension to consider in analyzing loss exposures is total dollar losses for all occurrences during a specific period, calculated by multiplying loss frequency by loss severity. Total dollar losses are a simplified way to combine frequency and severity distributions that have multiple possible outcomes. Expected total dollar losses can be projected by multiplying expected loss frequency by expected loss severity, and worst case scenarios can be calculated combining the frequency and severity distributions in the exhibit would be difficult given the total number of possible combinations. The risk analyst could make some simpler calculations to determine what the potential total dollar losses may be.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Number of losses | Probability |  | Dollar Losses | Probability |
| F0 | 0 | .03 | S1 | $100 | .30 |
| F1 | 1 | .05 | S2 | $250 | .25 |
| F2 | 2 | .08 | S3 | $500 | .20 |
| F3 | 3 | .10 | S4 | $683 | .15 |
| F4 | 4 | .15 | S5 | **$883** | .10 |
| F5 | 5 | .20 |  |  |  |
| F6 | 6 | .15 |  |  |  |
| F7 | 7 | .10 |  |  |  |
| F8 | 8 | .08 |  |  |  |
| F9 | **9** | .05 |  |  |  |
| F10 | 10 | .01 |  |  |  |

Combining the frequency and severity distributions in the exhibit would be difficult given the total number of possible combinations. The risk analyst could make some simpler calculations to determine what the potential total dollar losses may be. In this example, expected total dollar losses would be $1,877.93, and the worst case scenario could be calculated as $7,947.00, using F9 in the exhibit (F10 was not used, given its low probability).

Expected value = 4.9

Expected Total Dollar losses = 4.9 x $484.25 = 1,877.93 Frequency X Total dollar losses

Worst Case Total Dollar Losses = 9 X $883 = 7,947.00

**Timing**

**The fourth dimension to consider in analyzing loss exposures is timing of losses. This analysis requires considering when losses are likely to occur and when payment for those losses will likely be made. The timing dimension is significant because money held in reserve to pay for loss can earn interest until the actual payment is made. Whether a loss is counted when it is incurred or when it is paid is also significant for various accounting and tax reasons** that are beyond the scope of this discussion.

Funds to pay for property losses are generally dispersed relatively soon after the event occurs. In contrast, liability losses often involve long delays between occurrence of the adverse event, when an occurrence is recognized, the period of possible litigation, and the time when payment is actually made. The delay can span decades in losses involving environmental exposures or health risks.

**Data Credibility**

After analyzing the four dimensions of a loss exposure, the analysts then evaluates the credibility of the projections of loss frequency, loss severity, total dollar losses, and timing. The term “data credibility” refers to the level of confidence that available data can **accurately indicate future losses**.

Several factors may influence data credibility. Internally, changes in the way an organization operates, such as alterations to manufacturing processes or changes in data collection methods, may significantly reduce credibility of previously collected data. Externally, events such as natural catastrophes, large liability awards, or technological change not only alter the data that is collected in that time frame but also may cause shifts in the operating environment that render previously collected data less credible.

**This leaves risk professionals with a dilemma: Is it better to use older data, which is accurate but may have been generated in an environment that was substantially different from that of the period for which they are trying to predict, or to use more recent data and sacrifice some accuracy to maintain the integrity of the environment**?